THE UNIVERSITY OF AUCKLAND
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Te Whare Wānanga o Tāmaki Makaurau

Extending Collateralised Debt Obligation Portfolio Optimisation

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Background

Collateralised Debt Obligations are a form of asset backed financial security. A CDO is:

- Based upon an underlying portfolio of fixed income assets.
- Represents an agreement between the Investors in the CDO and the Asset Managers.
- As the assets in the underlying portfolio repay their debt the managers of the CDO distribute these funds to investors.

CDOs differentiate themselves from generic portfolios in the way that such payments are made.

- Investors in a CDO are ranked into separate groups called tranches.
- When the CDO makes a distribution (i.e. payment) to investors they are paid out in order of tranche seniority. T
- his means that the lower tranches absorb all of the initial risk in the portfolio. As can be seen in Fig 1

Introduction

A linear optimisation model has been developed to optimise a CDO portfolio. We seek to incorporate the Moody's Diversity Score into the model, and to find a form of the model suitable for multi-objective optimisation.

Diversity Score

$$DS = \sum_{k\in K} F\left(\sum_{j\in J_k} \min\left\{\frac{\sum_{\hat{j}\in J} z_j \sum_{i\in I_j} x_i v_i}{\sum_{i\in \mathrm{I}} x_i v_i}, 1\right\}\right)$$

The expression for the Diversity Score is highly non-linear and represents a relatively complex calculation process. We use binary variables and a number of substitutions to write the Diversity Score as a System of Linear expressions.

Step Function

The function F in the description of the Diversity Score is a step function. This is also non-linear, which means that we again had to alter its representation to incorporate it into the linear model. We used a variant of a method known as Special Ordered Sets to achieve this mapping and were able to do so in only linear expressions.

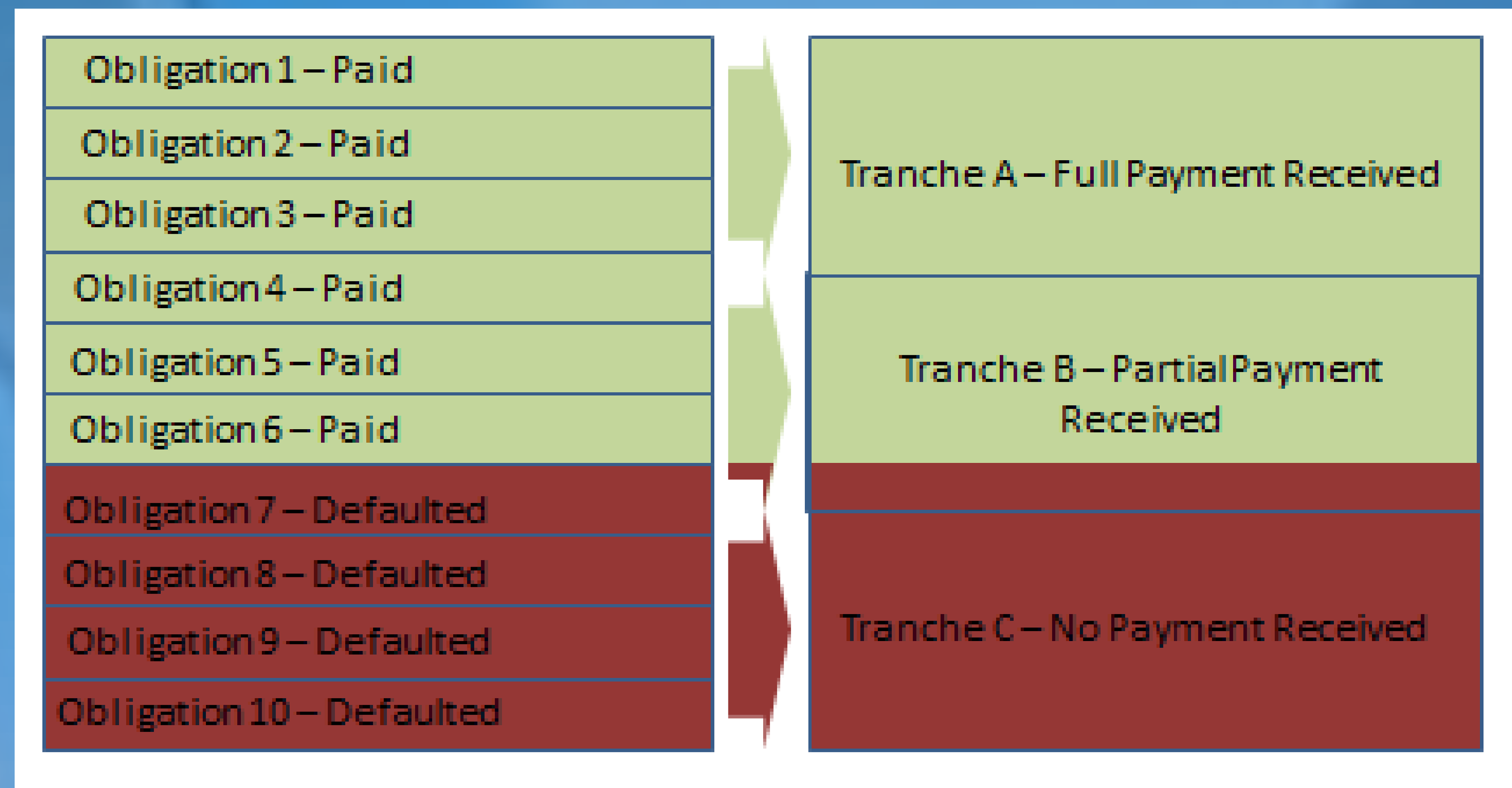

Figure 1: Depiction of the CDO payment procedure

Substitution Method

The substitution method is based upon the fact that multiplication with a binary variable.

In order to perform a multiplication we replace variables in the problem. For example the multiplication:

$$a\sum x_i$$

We replace x_i with a_i, and use the constraints as below to ensure that a_i takes the value of a if and only if x_i = 1.

$$a_i \le a + Mx_i$$
$$a_i \ge a - (1 - M)x_i$$
$$a_i \le a$$

System of Inequalities to perform a substitution, where M is a large number.

Multi-Objective Optimisation

There are four measurements of portfolio performance that have currently been incorporated into the model.

- Par Coverage Value (a measure of net realisable value of the portfolio)
- The Diversity Score (a measure of portfolio diversification),
- Weighted Average Spread (the interest rate generated by the portfolio)
- and Weighted Average Rating Factor (a measure of risk)

We find a linear form of the Weighted Average Values using a similar substitution to that given in the substitution method.

This allows us to define the objective function

$$\textbf{Max} \quad \textbf{Par Value} + \textbf{Diversity Score} + \boldsymbol{s}^T\hat{\boldsymbol{x}} + \boldsymbol{r}^T\hat{\boldsymbol{x}}$$

Where Weighted Average Spread = $\boldsymbol{s}^T\hat{\boldsymbol{x}}$
Weighted Average Rating Factor = $\boldsymbol{r}^T\hat{\boldsymbol{x}}$

Conclusions

- We have successfully incorporated the Diversity Score into the linear optimisation model
- The model has been altered so that we may perform multi-objective optimisation on a CDO portfolio
- The optimisation model has provided results that reflect improvements on current practice

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